SL Paper 1

The Venn diagram below shows events A and B where P(A) = 0.3, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.1$. The values m, n, p and q are

probabilities.



a(i) (a) d (i) Write down the value of n.

(ii) Find the value of m, of p, and of q.

b. Find P(B').

A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble and then selects a second marble.

a.	Write down the probability that the first marble Anna selects is red.	[1]
b.	Find the probability that Anna selects two red marbles.	[2]
c.	Find the probability that one marble is red and one marble is blue.	[3]

Consider the events A and B, where P(A) = 0.5, P(B) = 0.7 and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.

[4]

[2]



a(i), Wijiaen d (iii) n the value of

- (i) *p*;
 (ii) *q*;
 (iii) *r*.
 b. Find the value of P(*A*|*B'*).
- c. Hence, or otherwise, show that the events A and B are **not** independent.

Let
$$f(x)=rac{1}{2}x^2+kx+8$$
 , where $k\in\mathbb{Z}$.

a.	Find the values of k such that $f(x) = 0$ has two equal roots.	[4]
b.	Each value of k is equally likely for $-5 \le k \le 5$. Find the probability that $f(x) = 0$ has no roots.	[4]

Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

a. On the diagram below, shade the region representing P(X > 105).



- b. Given that $\mathrm{P}(X < d) = \mathrm{P}(X > 105)$, find the value of d.
- c. Given that P(X > 105) = 0.16 (correct to two significant figures), find P(d < X < 105).

[2]

[3]

[1]

[2]

[2]

The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	р	160

a. Find p.

b(i) Tahe (ii) mulative frequency diagram is given below.



Use the diagram to estimate

- (i) the 80th percentile;
- (ii) the interquartile range.

The letters of the word PROBABILITY are written on 11 cards as shown below.



Two cards are drawn at random without replacement.

[2]

[5]

Let *A* be the event the first card drawn is the letter A.

Let B be the event the second card drawn is the letter B.

a. Find P(A). b. Find P(B|A). c. Find $P(A \cap B)$.

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



[1]

[2]

[3]

[2]

Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

a.	Write down $\mathrm{P}(X>107).$	[1]
b.	Find $\mathrm{P}(100 < X < 107).$	[3]
c.	Find $\mathrm{P}(93 < X < 107).$	[2]

A four-sided die has three blue faces and one red face. The die is rolled.

Let *B* be the event a blue face lands down, and *R* be the event a red face lands down.

- a. Write down
 - (i) P(*B*);
 - (ii) P(R).
- b. If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the [2] following tree diagram, where *p*, *s*, *t* are probabilities.



Find the value of *p*, of *s* and of *t*.

c. Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 [3] and rolls one more time. Let *X* be the total score obtained.

[5]

[4]

- Show that $P(X = 3) = \frac{3}{16}$. Find P(X = 2). (i)
- (ii)

d. (i) Construct a probability distribution table for *X*.

- (ii) Calculate the expected value of *X*.
- e. If the total score is 3, Guiseppi wins \$10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$10.

Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

a.	Find	the probability that	[5]
		(i) none of the marbles are green;	
		(ii) exactly one marble is green.	
b.	Find	the expected number of green marbles drawn from the jar.	[3]
c.	(i)	Write down the probability that the marble is drawn from jar B.	[2]
	(ii)	Given that the marble was drawn from jar B, write down the probability that it is red.	
d.	Giver	n that the marble is red, find the probability that it was drawn from jar A.	[6]

Adam travels to school by car (*C*) or by bicycle (*B*). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



a.	Find the value of <i>p</i> .	[2]
b.	Find the probability that Adam will travel by car and be late for school.	[2]
c.	Find the probability that Adam will be late for school.	[4]
d.	Given that Adam is late for school, find the probability that he travelled by car.	[3]
e.	Adam will go to school three times next week.	[4]
	Find the probability that Adam will be late exactly once.	

[5]

a(i) Bag (A) contains three white balls and four red balls. Two balls are chosen at random without replacement.

(i) Copy and complete the following tree diagram.



(ii) Find the probability that two white balls are chosen.

b. Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B. Find the probability that the two balls are white.

c. Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B. Given that both balls are white, find the probability that they were chosen from bag A.

The following is a cumulative frequency diagram for the time *t*, in minutes, taken by 80 students to complete a task.



a.	Find the number of students who completed the task in less than 45 minutes.	[2]
b.	Find the number of students who took between 35 and 45 minutes to complete the task.	[3]
c.	Given that 50 students take less than k minutes to complete the task, find the value of k .	[2]

Let A and B be independent events, where $\mathrm{P}(A)=0.6$ and $\mathrm{P}(B)=x$.



[1] [4]

- (ii) find $P(A \cap B)$.
- c. Hence, explain why *A* and *B* are not mutually exclusive.

A discrete random variable X has the following probability distribution.

x	0	1	2	3
P(X=x)	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	р

a. Find p.

b. Find E(X).

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	Tennis	Hockey
Female	5	3	3
Male	4	2	3

a(i) Omd (si)udent is selected at random.

- (i) Calculate the probability that the student is a male or is a tennis player.
- (ii) Given that the student selected is female, calculate the probability that the student does not play football.
- b. Two students are selected at random. Calculate the probability that neither student plays football.

Let
$$\boldsymbol{A} = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$$
 and $\boldsymbol{B} = \begin{pmatrix} -4 & 0 \\ 5 & 1 \end{pmatrix}$

a. Find *AB* .

b. Given that X - 2A = B, find X.

[3]

[3]

[4]

[3]

[3]

$$\mathrm{P}(X=x)=rac{x^2}{14}, x\in \left\{1,2,k
ight\}, \mathrm{where} k>0$$

a. Write down $P(X = 2)$.	[1]

- b. Show that k = 3.
- c. Find E(X).

The following table shows the probability distribution of a discrete random variable X.

x	0	2	5	9
P(X = x)	0.3	k	2k	0.1

a. Find the value of k.

b. Find E(X).

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



a.i. Find the value of p;

a.ii.Find the value of q.

b. A girl is selected at random. Find the probability that she takes economics but not history.

[2]

[2]

[2]

[4]

[2]

[3]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



The following box-and-whisker plot represents the examination scores of a group of students.



a. Write down the median score.

The range of the scores is 47 marks, and the interquartile range is 22 marks.

- b. Find the value of
 - (i) *c*;
 - (ii) *d*.

A data set has *n* items. The sum of the items is 800 and the mean is 20.

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

a. Find <i>n</i> .	[2]
b.i.Write down the value of the new mean.	[1]
b.iiFind the value of the new variance.	[3]

There are 10 items in a data set. The sum of the items is 60.

The variance of this data set is 3. Each value in the set is multiplied by 4.

a. Find the mean.

- b. (i) Write down the value of the new mean.
 - (ii) Find the value of the new variance.

José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event "he misses his bus" and F the event "he is late for school".

The information above is shown on the following tree diagram.

[4]

[2]



a(i) Einod(ii).

- (i) $P(E \cap F)$;
- (ii) P(F).

b(i) **Eind** (ii) e probability that

- (i) José misses his bus and is not late for school;
- (ii) José missed his bus, given that he is late for school.
- c. The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

Copy and complete the probability distribution table.

X (cost in euros)	0	3	6
P (X)	$\frac{1}{9}$		

d. The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

Find the expected cost for José for both days.

In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.

[4]

[5]

[3]

[2]



a(i),(i) and fit e down the value of s.

- (ii) Find the value of q.
- (iii) Write down the value of p and of r.

b(i) (and (ii), student is selected at random. Given that the student takes music, write down the probability the student takes art.

- (ii) Hence, show that taking music and taking art are not independent events.
- c. Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student [4] takes **only** art.

A running club organizes a race to select girls to represent the club in a competition.

The times taken by the group of girls to complete the race are shown in the table below.

Time <i>t</i> minutes	$10 \le t < 12$	$12 \le t < 14$	$14 \le t < 20$	$20 \le t < 26$	$26 \le t < 28$	$28 \le t < 30$
Frequency	50	20	р	40	20	20
Cumulative Frequency	50	70	120	q	180	200

- a. Find the value of p and of q.
- b. A girl is chosen at random.
 - (i) Find the probability that the time she takes is less than 14 minutes.
 - (ii) Find the probability that the time she takes is at least 26 minutes.
- c. A girl is selected for the competition if she takes less than x minutes to complete the race.

[5]

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[4]

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[4]

Given that 40% of the girls are not selected,

- (i) find the number of girls who are not selected;
- (ii) find x.
- d. Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times [4]

taken by these girls are shown in the cumulative frequency diagram below.



(i) Write down the number of girls who were allowed another chance.

(ii) Find the percentage of the whole group who were selected.

The following Venn diagram shows the events A and B, where P(A) = 0.4, $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.1$. The values p and q are probabilities.



- a. (i) Write down the value of q.
 - (ii) Find the value of p.
- b. Find P(B).

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

- a. Find P(B).
- b. Find $P(A \cup B)$.

The following diagram shows a board which is divided into three regions A, B and C.

A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	А	В	С
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

a. Find the probability that the dart does **not** hit the board.

b. The contestant scores points as shown in the following table.

Region	А	В	С	Does not hit the board
Points	0	q	10	-3

Given that the game is fair, find the value of q.

The cumulative frequency curve below represents the marks obtained by 100 students.



[3]

[4]

[2]

[4]



- a. Find the median mark.
- b. Find the interquartile range.

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.



[2]

[3]

[2]

[4]

- a. Find the value of the interquartile range.
- b. One student sent k text messages, where k > 11. Given that k is an outlier, find the least value of k.

The random variable X has the following probability distribution.

x	1	2	3
P(X = x)	S	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find q .

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.

The following Venn diagram shows the events "own a laptop" and "own a tablet".

The values p, q, r and s represent numbers of students.



[5]

[4]

[4]

A student is selected at random from the class.

Two students are randomly selected from the class. Let L be the event a "student owns a laptop".

- a. (i) Write down the value of p.
 - (ii) Find the value of q.
 - (iii) Write down the value of r and of s.
- b. (i) Write down the probability that this student owns a laptop.
 - (ii) Find the probability that this student owns a laptop or a tablet but not both.
- c. (i) Copy and complete the following tree diagram. (Do not write on this page.)



(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop.

Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

a.	. Copy and complete the table below to show all nine equally likely outcomes.	[2]
	3,9	
	3,10	
	3,10	
b.	• Let <i>S</i> be the sum of the numbers on the two cards. Find the probability of each value of <i>S</i> .	[2]
c.	Find the expected value of S.	[3]
d.	Anna plays a game where she wins 50 if S is even and loses 30 if S is odd.	[3]
	Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.	

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

a.i. Write down the independent variable.						[1]
a.ii.Write down the boiling temperature of	the liquid.					[1]
b. Jim describes the correlation as very strong. Circle the value below which best represents the correlation coefficient.						[2]
	0.992	0.251	0	-0.251	-0.992	
c. Jim's model is $d=-2.24t+105$, for	$0\leqslant t\leqslant 20.$	Use his mod	lel to pred	ict the decrease	in temperature for any 2 minute interval.	[2]

Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$. If Bill wins the first game, the probability that he wins the second game is $\frac{5}{6}$.

If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

a. Copy and complete the following tree diagram. (Do not write on this page.)



b. Find the probability that Bill wins the first game and Andrea wins the second game.[2]c. Find the probability that Bill wins at least one game.[4]d. Given that Bill wins at least one game, find the probability that he wins both games.[5]

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.



Let X be the sum of the scores on the two dice.

a(i), (ii) and iii. P(X = 6).

- (ii) Find P(X > 6).
- (iii) Find P(X = 7 | X > 6).

b. Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.

If the sum is greater than 6, she **wins** 1 point.

If the sum is less than 6, she **loses** *k* points.

Find the value of *k* for which the game is fair.

There are nine books on a shelf. For each book, x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.



- a. Write down the possible minimum and maximum values of r.
- b. Given that r = 0.95, which of the following diagrams best represents the data.



c. For the data in diagram D, which two of the following expressions describe the correlation between x and y?

[2]

[2] [1] A box contains 100 cards. Each card has a number between one and six written on it. The following table shows the frequencies for each number.

[2]

[5]

Number	1	2	3	4	5	6
Frequency	26	10	20	k	29	11

a. Calculate the value of *k*.

b(i) **Taind** (ii).

- (i) the median;
- (ii) the interquartile range.

A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the

festival.



The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours.

a.i. Find the median number of hours worked by the employees.	[2]
a.ii.Write down the number of employees who worked 50 hours or less.	[1]
b.i.Find the amount of money an employee earned for working 40 hours;	[1]
b.iiFind the amount of money an employee earned for working 43 hours.	[3]
c. Find the number of employees who earned \pounds 200 or less.	[3]
d. Only 10 employees earned more than $\pounds k$. Find the value of k .	[4]

Events A and B are such that $\mathrm{P}(A)=0.3$, $\mathrm{P}(B)=0.6$ and $\mathrm{P}(A\cup B)=0.7$.



The values q, r, s and t represent probabilities.

- a. Write down the value of t.
- b(i) (a) is how that r = 0.2.
 - (ii) Write down the value of q and of s.
- c(i) (ai) d (i) Write down P(B').
 - (ii) Find P(A|B').

In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

a.	(i)	Find the number of boys who play both sports.	[3]
	(ii)	Write down the number of boys who play only rugby.	
b.	One	boy is selected at random.	[4]
	(i)	Find the probability that he plays only one sport.	
	(ii)	Given that the boy selected plays only one sport, find the probability that he plays rugby.	
c.	Let A	be the event that a boy plays football and <i>B</i> be the event that a boy plays rugby.	[2]
	Expla	ain why A and B are not mutually exclusive.	
d.	Show	that A and B are not independent.	[3]

[1]

[3]

[3]

The diagram below shows the probabilities for events A and B, with P(A') = p.



- a. Write down the value of p.
- b. Find P(B).
- c. Find P(A'|B).

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs.

The taxicabs are randomly assigned by the company.

The probability that a taxicab is yellow is 0.4.

The probability that a taxicab is a Fiat is 0.3.

The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ .

а	1	2	
$\mathbb{P}(A = a)$	$\cos \theta$	$2\cos 2\theta$	

a. Show that
$$\cos \theta = \frac{3}{4}$$
.

b. Given that $\tan \theta > 0$, find $\tan \theta$.

c. Let
$$y = \frac{1}{\cos x}$$
, for $0 < x < \frac{\pi}{2}$. The graph of ybetween $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

[6]

[3]

[1]

[3]

A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.



- a. Find the range of the lengths of **all** 200 fish.
- b. Four cumulative frequency graphs are shown below.



Which graph is the best representation of the lengths of the female fish?

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

a. Complete the following tree diagram.

[3] [2]



b. Find the probability that exactly one of the selected balls is green.

The random variable X has the following probability distribution, with $\mathrm{P}(X>1)=0.5$.

x	0	1	2	3
P(X = x)	p	q	r	0.2

- a. Find the value of r.
- b. Given that E(X) = 1.4, find the value of p and of q.

A school collects cans for recycling to raise money. Sam's class has 20 students.

The number of cans collected by each student in Sam's class is shown in the following stem and leaf diagram.

Stem	Leaf	Key: 3 1 represents 31 cans
2	0, 1, 4, 9, 9	
3	1, 7, 7, 7, 8, 8	
4	1, 2, 2, 3, 5, 6, 7, 8	
5	0	

The following box-and-whisker plot also displays the number of cans collected by students in Sam's class.



There are 80 students in the school.

[6]



The mean number of cans collected is 39.4. The standard deviation is 18.5. Each student then collects 2 more cans.

a.	Find	the median number of cans collected.	[2]
b.	(i)	Write down the value of <i>a</i> .	[3]
	(ii)	The interquartile range is 14. Find the value of <i>b</i> .	
c.	Sam	's class collected 745 cans. They want an average of 40 cans per student.	[3]
	How	many more cans need to be collected to achieve this target?	
d.	The	students raise \$0.10 for each recycled can.	[5]
	(i)	Find the largest amount raised by a student in Sam's class.	

(ii) The following cumulative frequency curve shows the amounts in dollars raised by all the students in the school. Find the percentage of students in the school who raised more money than anyone in Sam's class.





(ii) Write down the new standard deviation.

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$. If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$. If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

a. Copy and complete the following tree diagram.



b. Find the probability that Pablo leaves home before 07:00 and is late for work.

[2]

e. Two days next week Pablo will drive to work. Find the probability that he will be late at least once.



The weekly wages (in dollars) of 80 employees are displayed in the cumulative frequency curve below.

a(i) (and (i) Write down the median weekly wage.

(ii) Find the interquartile range of the weekly wages.

b(i),**T(h)e (bii)**x-and-whisker plot below displays the weekly wages of the employees.

[3]

